

# VIDYA BHAWAN BALIKA VIDYA PITH

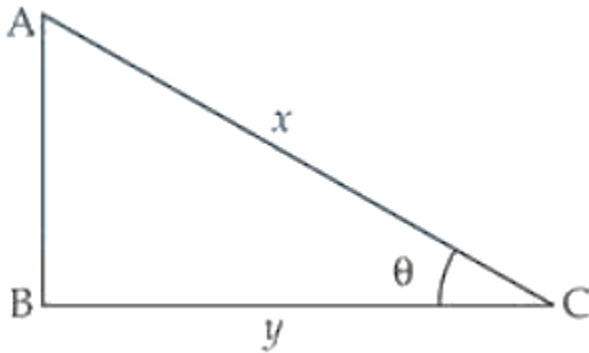
शक्तिउत्थानआश्रमलखीसरायबिहार

Class-12 Sub-.Maths

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25. If the sum of the lengths of the hypotenuse and a side of a right angled triangle is given, show that the area of the triangle is maximum when the angle between them is  $\pi/3$ .

Solution:



Let  $\Delta ABC$  be the right-angled triangle in

which  $\angle B = 90^\circ$

Let  $AC = x$ ,  $BC = y$

So,  $AB = \sqrt{x^2 + y^2}$

$\angle ACB = \theta$

Let  $z = x + y$  (given)

Now, the area of  $\Delta ABC = \frac{1}{2} \times AB \times BC$

$$\Rightarrow A = \frac{1}{2} y \cdot \sqrt{x^2 - y^2} \Rightarrow A = \frac{1}{2} y \cdot \sqrt{(Z - y)^2 - y^2}$$

Squaring both sides, we get

$$A^2 = \frac{1}{4} y^2 [(Z - y)^2 - y^2] \Rightarrow A^2 = \frac{1}{4} y^2 [Z^2 + y^2 - 2Zy - y^2]$$

$$\text{So, } P = \frac{1}{4} y^2 [Z^2 - 2Zy] \Rightarrow P = \frac{1}{4} [y^2 Z^2 - 2Zy^3] \quad [A^2 = P]$$

Differentiating both sides w.r.t.  $y$  we get

$$\frac{dP}{dy} = \frac{1}{4} [2yZ^2 - 6Zy^2] \quad \dots(i)$$

For local maxima and local minima,  $\frac{dP}{dy} = 0$

$$\therefore \frac{1}{4} (2yZ^2 - 6Zy^2) = 0$$

$$\frac{2yZ}{4} (Z - 3y) = 0 \Rightarrow yZ(Z - 3y) = 0$$

$$yZ \neq 0 \quad (\because y \neq 0 \text{ and } Z \neq 0)$$

$$\therefore Z - 3y = 0$$

$$y = \frac{Z}{3} \Rightarrow y = \frac{x + y}{3} \quad (\because Z = x + y)$$

$$3y = x + y \Rightarrow 3y - y = x \Rightarrow 2y = x$$

$$\frac{y}{x} = \frac{1}{2} \Rightarrow \cos \theta = \frac{1}{2}$$

$$\text{Thus, } \theta = \frac{\pi}{3}$$

Differentiating eq. (i) w.r.t.  $y$ , we have  $\frac{d^2P}{dy^2} = \frac{1}{4} [2Z^2 - 12Zy]$

$$\frac{d^2P}{dy^2} \text{ at } y = \frac{Z}{3} = \frac{1}{4} \left[ 2Z^2 - 12Z \cdot \frac{Z}{3} \right]$$

$$= \frac{1}{4} [2Z^2 - 4Z^2] = \frac{-Z^2}{2} < 0 \text{ Maxima}$$

$$\Rightarrow A = \frac{1}{2} y \cdot \sqrt{x^2 - y^2} \Rightarrow A = \frac{1}{2} y \cdot \sqrt{(Z - y)^2 - y^2}$$

Squaring both sides, we get

$$A^2 = \frac{1}{4} y^2 [(Z - y)^2 - y^2] \Rightarrow A^2 = \frac{1}{4} y^2 [Z^2 + y^2 - 2Zy - y^2]$$

$$\text{So, } P = \frac{1}{4} y^2 [Z^2 - 2Zy] \Rightarrow P = \frac{1}{4} [y^2 Z^2 - 2Zy^3] \quad [A^2 = P]$$

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$$= \frac{1}{4} [2Z^2 - 4Z^2] = \frac{-Z^2}{2} < 0 \text{ Maxima}$$

Therefore, the area of the given triangle is maximum when the angle between its hypotenuse and a side is  $\pi/3$ .

**26. Find the points of local maxima, local minima and the points of inflection of the function  $f(x) = x^5 - 5x^4 + 5x^3 - 1$ . Also find the corresponding local maximum and local minimum values.**

**Solution:**

$$\text{Given, } f(x) = x^5 - 5x^4 + 5x^3 - 1$$

Differentiating the function,

$$f'(x) = 5x^4 - 20x^3 + 15x^2$$

For local maxima and local minima,  $f'(x) = 0$

$$\begin{aligned} 5x^4 - 20x^3 + 15x^2 &= 0 \Rightarrow 5x^2(x^2 - 4x + 3) = 0 \\ \Rightarrow 5x^2(x^2 - 3x - x + 3) &= 0 \Rightarrow x^2(x - 3)(x - 1) = 0 \\ \therefore x &= 0, x = 1 \text{ and } x = 3 \end{aligned}$$

$$\text{Now } f''(x) = 20x^3 - 60x^2 + 30x$$

$\Rightarrow f''(x)_{\text{at } x=0} = 20(0)^3 - 60(0)^2 + 30(0) = 0$  which is neither maxima nor minima.

$\therefore f(x)$  has the point of inflection at  $x = 0$

$$\begin{aligned} f''(x)_{\text{at } x=1} &= 20(1)^3 - 60(1)^2 + 30(1) \\ &= 20 - 60 + 30 = -10 < 0 \text{ Maxima} \end{aligned}$$

$$\begin{aligned} f''(x)_{\text{at } x=3} &= 20(3)^3 - 60(3)^2 + 30(3) \\ &= 540 - 540 + 90 = 90 > 0 \text{ Minima} \end{aligned}$$

The maximum value of the function at  $x = 1$

$$\begin{aligned} f(x) &= (1)^5 - 5(1)^4 + 5(1)^3 - 1 \\ &= 1 - 5 + 5 - 1 = 0 \end{aligned}$$

The minimum value at  $x = 3$  is

$$\begin{aligned} f(x) &= (3)^5 - 5(3)^4 + 5(3)^3 - 1 \\ &= 243 - 405 + 135 - 1 \\ &= 378 - 406 = -28 \end{aligned}$$

Therefore, the function has its maxima at  $x = 1$  and the maximum value = 0 and its has minimum value at  $x = 3$  and its minimum value is -28.

**27. A telephone company in a town has 500 subscribers on its list and collects fixed charges of Rs 300/- per subscriber per year. The company proposes to increase the annual subscription and it is believed that for every increase of Rs 1/- one subscriber will discontinue the service. Find what increase will bring maximum profit?**

**Solution:**

Let's consider that the company increases the annual subscription by Rs  $x$ .

So,  $x$  is the number of subscribers who discontinue the services.

$$\text{Total revenue, } R(x) = (500 - x)(300 + x)$$

$$= 150000 + 500x - 300x - x^2$$

$$= -x^2 + 200x + 150000$$

Differentiating both sides w.r.t.  $x$ , we get  $R'(x) = -2x + 200$

For local maxima and local minima,  $R'(x) = 0$

$$-2x + 200 = 0 \Rightarrow x = 100$$

$$R''(x) = -2 < 0 \text{ Maxima}$$

So,  $R(x)$  is maximum at  $x = 100$

Therefore, in order to get maximum profit, the company should increase its annual subscription by Rs 100.